CHAPTER 10

TOPICS IN ANALYTIC GEOMETRY (INTRO TO CONICS)

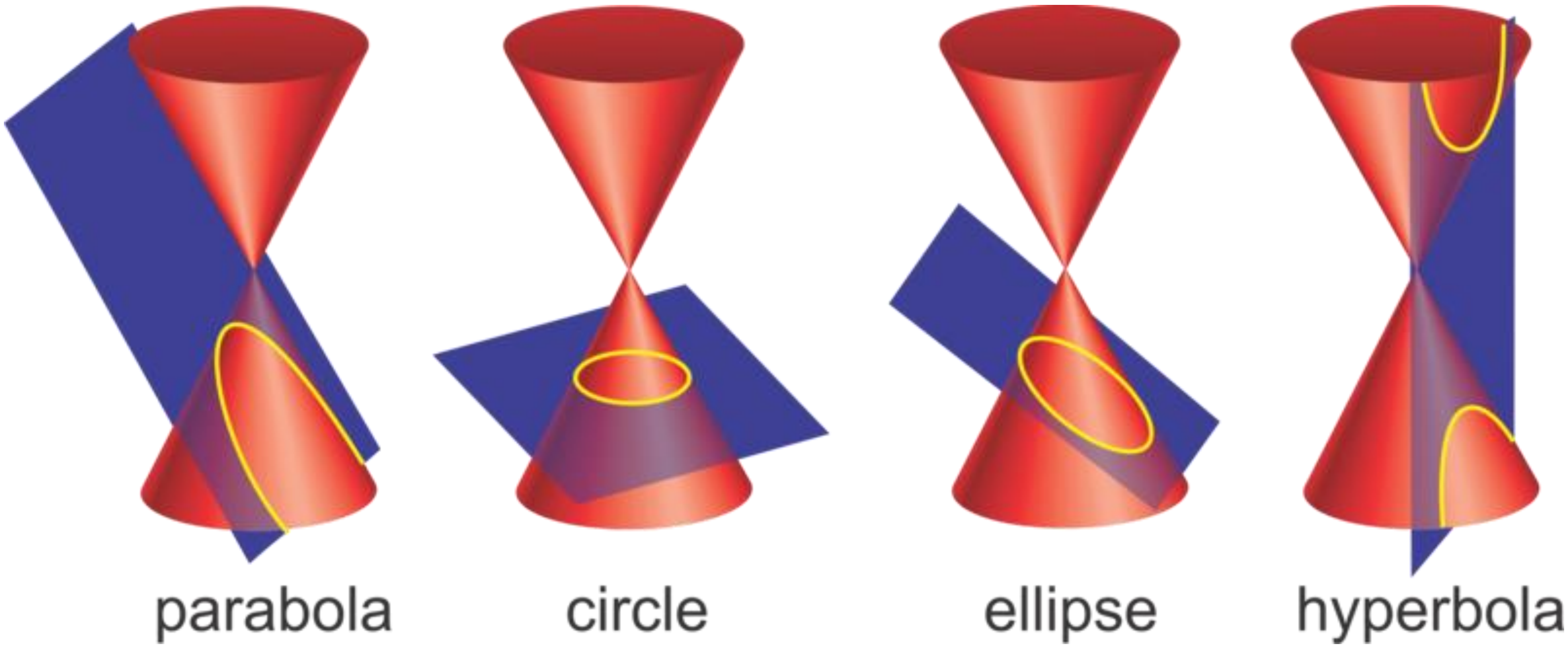
10A PARABOLAS

10B ELLIPSES / CIRCLES

10C HYPERBOLAS

CONICS

Conic sections are four shapes; parabolas, circles, ellipses, and hyperbolas, created from the intersection of a plane with a cone or two cones.



IN MATH

- Discovered by the Greeks around 600 to 300 BC.
- Not until the 17^{th} century did the applicability of conics become apparent.
- Conics played a prominent role in the early development of calculus.

IN REAL LIFE

- Conics are used as models in construction, planetary orbits, navigation, and projectile motion.
- Parabolas are used to model the cables of the Golden Gate Bridge
- An ellipse is used to model the orbit of Halley's Comet as well as the orbits of planets as they move about the sun.
- Hyperbolas are used in long distance radio navigation for aircraft and ships.

IN CAREERS

- There are many careers that use conics and other topics in analytic geometry.
- Home Contractor
- Civil Engineer
- Artist
- Astronomer

PRACTICE: COMPLETE THE SQUARE!

A Perfect Square Trinomial is a Trinomial that will FACTOR into two identical binomials, so you can write it as a quantity squared.

Example: $x^2 + 6x + 9 \quad = \quad (x+3)(x+3) \quad = \quad (x+3)^2$

Complete the Square

1. $x^2 - 8x +$ ________

2. $x^2 + 5x +$ ________

3. $2x^2 - 4x +$ ________

10A: PARABOLAS

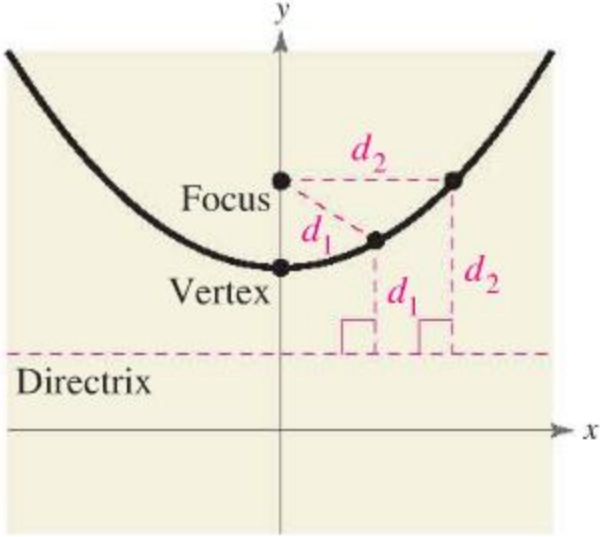
In Section 2.1, you learned that the graph of the quadratic function

$$f(x) = ax^2 + bx + c$$

is a parabola that opens upward or downward. The following definition of a parabola is more general in the sense that it is independent of the orientation of the parabola.

Definition of Parabola

A **parabola** is the set of all points (x, y) in a plane that are equidistant from a fixed line (**directrix**) and a fixed point (**focus**) not on the line.



VERTEX: The midpoint between the focus and the directrix.

AXIS: The line passing through the focus and the vertex. A parabola is symmetric with respect to its axis.

Standard Equation of a Parabola

The **standard form of the equation of a parabola** with vertex at (h, k) is as follows.

$$(x - h)^2 = 4p(y - k), \quad p \neq 0 \qquad \text{Vertical axis, directrix: } y = k - p$$

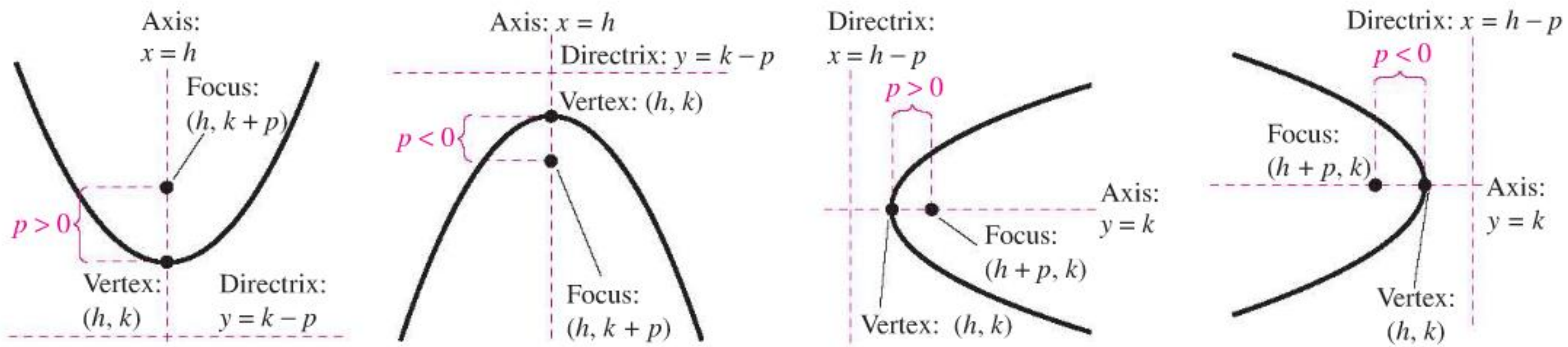
$$(y - k)^2 = 4p(x - h), \quad p \neq 0 \qquad \text{Horizontal axis, directrix: } x = h - p$$

The focus lies on the axis p units (*directed distance*) from the vertex. If the vertex is at the origin $(0, 0)$, the equation takes one of the following forms.

$$x^2 = 4py \qquad \text{Vertical axis}$$

$$y^2 = 4px \qquad \text{Horizontal axis}$$

See Figure 10.12.

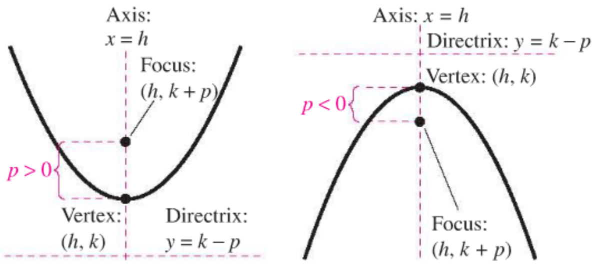


(a) $(x - h)^2 = 4p(y - k)$
Vertical axis: $p > 0$

(b) $(x - h)^2 = 4p(y - k)$
Vertical axis: $p < 0$

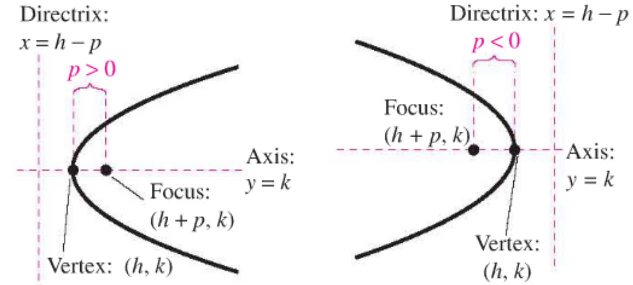
(c) $(y - k)^2 = 4p(x - h)$
Horizontal axis: $p > 0$

(d) $(y - k)^2 = 4p(x - h)$
Horizontal axis: $p < 0$



(a) $(x - h)^2 = 4p(y - k)$
Vertical axis: $p > 0$

(b) $(x - h)^2 = 4p(y - k)$
Vertical axis: $p < 0$



(c) $(y - k)^2 = 4p(x - h)$
Horizontal axis: $p > 0$

(d) $(y - k)^2 = 4p(x - h)$
Horizontal axis: $p < 0$

Vertex NOT at origin

Vertex AT origin

‘p’ is the distance from vertex to focus
Positive direction: +p
Negative direction: -p

Find Vertex, Focus, and Directrix

Given the equation of a parabola, find the Vertex, Focus, & Directrix

1. $y^2 = 16x$

2. $(x-2)^2 = -8(y+1)$

Vertex at the Origin

Find the equation of the following parabolas:

3. Vertex at origin
   Focus (0, 2)

4. Vertex at origin
   Focus (–3, 0)

Find Vertex, Focus, & Directrix

5. $y = -\frac{1}{2}x^2 - x + \frac{1}{2}$

Original Eq. (what kind of parabola?)

Multiply by -2

Add 1 to each side

Complete the square

Combine like terms/Factor

Standard Form

Vertex (h, k)

Focus (h, k+p)

Directrix y = k – p

Find Standard Form Given Characteristics

6. Find the standard form of the equation of a parabola with Vertex (1, 2) and Focus (1, 5).

Think. Make sketch. (smiley)
Vertical Axis

h = 1, k = 2, p = 3 (p = 5 – 2)

STANDARD FORM

Multiply

Add 24 to both sides

Divide by 12

QUADRATIC FORM